

# On the minimum mass ratio of W UMa binaries

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## ABSTRACT

Using Eggleton’s stellar evolution code, we study the minimum mass ratio ( $q_{\min}$ ) of W Ursae Majoris (W UMa) binaries that have different primary masses. It is found that the minimum mass ratio of W UMa binaries decreases with increasing mass of the primary if the primary’s mass is less than about  $1.3M_{\odot}$ , and above this mass the ratio is roughly constant. By comparing the theoretical minimum mass ratio with the observational data, it is found that the existence of low- $q$  systems can be explained by the different structure of the primaries with different masses. This suggests that the dimensionless gyration radius ( $k_1^2$ ) and thus the structure of the primary is very important in determining the minimum mass ratio. In addition, we investigate the mass loss during the merging process of W UMa systems and calculate the rotation velocities of the single stars formed by the merger of W UMa binaries due to tidal instability. It is found that in the case of the conservation of mass and angular momentum, the merged single stars rotate with a equatorial velocity of about  $588 \sim 819 \text{ km s}^{-1}$ , which is much larger than their break-up velocities ( $v_b$ ). This suggests that the merged stars should extend to a very large radius (3.7~5.3 times the radii of the primaries) or W UMa systems would lose a large amount of mass (21~33 per cent of the total mass) during the merging process. If the effect of magnetic braking is considered, the mass loss decreases to be 12~18 per cent of their total masses. This implies that the significant angular momentum and mass might be lost from W UMa systems in the course of the merging process, and this kind of mass and angular momentum

loss might be driven by the release of orbital energy of the secondaries, which is similar to common-envelope evolution.

**Key words:** instabilities – binaries: close – blue stragglers – stars: evolution – stars: rotation

## 1 INTRODUCTION

W Ursae Majoris (W UMa) binaries are eclipsing variables in which two components are in contact or overflowing their Roche limiting surfaces. The components of W UMa binaries share a common convective envelope. In general, W UMa systems have total system masses  $1M_{\odot} \leq (M_1 + M_2) \leq 3M_{\odot}$  and orbital periods between 0.22 and 1 days (Gazeas & Niarchos 2006). They are very common systems and can be discovered in field, open clusters, and globular clusters (Kaluzny & Rucinski 1993; Rucinski 1994, 1998, 2000). There is at least one W UMa binary for every 500 main sequence stars in the solar neighborhood (Rucinski 2002, 2006). Rucinski (1994) gave the relative frequency of occurrence of one W UMa system per  $275 \pm 75$  ordinary dwarfs in open clusters. In globular clusters, the relative frequency of occurrence of W UMa systems was also found to be very high (Rucinski 2000).

Eggleton (2006) pointed out that many binaries of short period can be expected to evolve into contact, and there is only a small region in the initial orbital period and mass ratio plane where it does not contact if the Roche-lobe overflow starts while the primary is still in the main sequence band. W UMa systems have the least amounts of angular momentum that binaries made of main-sequence components can have, so they are important sources for testing the angular momentum evolution of binaries (Rucinski 2000; Selam 2004). More importantly, W UMa binaries can be used to study Galactic structure because they have high spatial frequency of occurrence, ease of detection and provide an absolute magnitude calibration (Rucinski 1997). In addition, W UMa systems are the possible progenitors of blue stragglers/FK Comae-type (FK Com) stars (Qian et al. 2005). Some blue stragglers in star clusters are likely formed from the merger of W UMa binaries (Lombardi et al. 2002). The investigation of the merger of W UMa binaries can help us to understand the formation theory on blue stragglers and FK Com-type stars.

Theoretical studies indicate that W UMa system would merge into a fast-rotating single star due to the tidal instability (i.e. Darwin’s instability) when the spin angular mo-

mentum of the system is more than a third of its orbital angular momentum (Hut 1980; Eggleton & Kiseleva-Eggleton 2001). The occurrence of the tidal instability in W UMa binaries determines the minimum mass ratio ( $q_{min}$ ) of these systems. W UMa binaries with mass ratio  $q = M_2/M_1 \leq q_{min}$  should not be observed since they have merged into fast-rotating stars within a tidal timescale (about  $10^3 - 10^4$  yr). Therefore, the minimum mass ratio is a very important parameter in investigating the evolution and the merger of W UMa systems.

The minimum mass ratio of W UMa binaries has been investigated by many authors (Rasio 1995; Li, Han & Zhang 2004, 2005; Arbutina 2007, 2009). If the rotation of the secondaries in W UMa systems is neglected, the minimum mass ratio of W UMa-type systems is derived to be of about 0.09 (Rasio 1995). If the rotation of the secondaries is taken into account and  $k_1^2 = k_2^2 = 0.06$  ( where  $k_1^2 = I_1/(M_1 R_1^2)$ ,  $k_2^2 = I_2/(M_2 R_2^2)$  are the dimensionless gyration radii for the primary and the secondary), the cutoff mass ratio of W UMa systems is derived to be of 0.076 (Li & Zhang 2006). If it is assumed that the primary is radiative main-sequence star ( $k_1^2 \approx 0.075$ ) and the secondary is fully convective star ( $k_2^2 \approx 0.205$ ), the theoretical minimum mass ratio is derived to be of about 0.094-0.109 (Arbutina 2007). These results predict that W UMa binaries with a mass ratio less than the minimum mass ratio should not be observed. However, the mass ratios of some observed W UMa binaries are smaller than the theoretical minimum mass ratio, such as V857 Her ( $q=0.065$  Qian et al. 2005), AW UMa ( $q=0.075$  Rucinski 1992) and SX Crv ( $q=0.079$  Zola et al. 2004). This can be explained by W UMa systems with slightly evolved primaries or differential rotation of their components (Rasio 1995; Li & Zhang 2006; Arbutina 2007). In order to remove the difference between observations and theoretical predictions, Arbutina (2009) obtained a theoretical minimum mass ratio of about 0.070-0.074 if the effects of rotation which increase the central concentration are included. However, little has been done to investigate the minimum ratio of W UMa systems that have different primary masses. Hurley, Pols & Tout (2000) showed that there is a difference in the structure of the main-sequence stars with different masses. Because the dimensionless gyration radius of the primary depends on the structure (Li, Han & Zhang 2005), this may introduce the structure of the primaries with different masses that may need to be taken into account in determining the minimum mass ratio of W UMa systems.

W UMa systems with  $q \leq q_{min}$  are unstable and undergo rapid merging, which might result in the formation of the rapidly rotating single stars (blue stragglers or FK Com, Webbink 1976; Stępień 1995; Li, Han & Zhang 2004, 2005). During the merger of a W UMa

system, the secondary enters the primary and it, together with the primary would spin up. Therefore, the equatorial velocity of the single star formed by the merger of W UMa binary would be larger than that of W UMa binaries which is of 100-200 km s<sup>-1</sup> (Selam 2004). However, the rotational velocities of the blue stragglers in field are found to be normal (Carney & Peterson 1981), while blue stragglers in M67 are rotating slowly compared with main-sequence stars (Mathys 1991). De Marco et al. (2005) measured the rotation velocities of five rapidly-rotating blue stragglers,  $v \sin i$  of about 120, 100, 225, 50, and 50 km s<sup>-1</sup>, but this information cannot constrain their origin as stellar collision or binary merger because of the lack of clear theoretical predictions. For FK Com ( $v \sin i \sim 160$  km s<sup>-1</sup>), its angular momentum is about 3 times smaller than that of the orbital motion in a typical W UMa binary (Rucinski 1990). Therefore, a large amount of mass and angular momentum must be lost from W UMa systems if some blue stragglers or FK Com stars are formed from the merger of W UMa systems. However, it is uncertain how much of the mass is lost during the merging process.

The purpose of this paper is to study the minimum mass ratio of W UMa systems and the mass loss during the merging process. Employing Eggleton's stellar evolution code, we study the structure of the primary with different mass, and then determine the minimum mass ratio of W UMa binaries. We compare the theoretical minimum mass ratio with the observational data and find that the existence of low- $q$  systems can be explained by the different structure of the primaries with different masses. We find that the structure of the primary is important in determining the minimum mass ratio of W UMa binaries. In addition, we investigate the mass loss during the merging process of W UMa systems. We find that W UMa systems should lose a large amount of mass to avoid these merged stars rotating faster than the break-up velocities. If the effect of the magnetic braking is considered, the angular momentum loss may be more efficient and the mass loss would decrease.

## 2 THE MINIMUM MASS RATIO

Rasio (1995) predicted that the dynamical stability limit should depend on the structure of W UMa systems with given masses  $M_1$  and  $M_2$ . We use a stellar evolution code to determine the dimensionless gyration radii ( $k^2$ ) of stars with different masses and ages, and study the effect of the interior structure (the dimensionless gyration radius) of components with different masses on the minimum mass ratio. This code is originally developed by Eggleton

Table 1. The physical parameters of W UMa binaries.

Stars	$q_{ph}$	$M_1$ ( $M_\odot$ )	$R_1$ ( $R_\odot$ )	$P$ (days)	$v_e$ ( $\text{km s}^{-1}$ )	$v_b$ ( $\text{km s}^{-1}$ )	$R_{ex}$ ( $R_\odot$ )	$\delta M$ ( $M_\odot$ )	$\delta M_{mb}$ ( $M_\odot$ )	References
AW UMa	0.078	1.79	1.9	0.4387	819.31	423.85	9.73	0.645	0.349	(1)
SX Crv	0.0787	1.246	1.347	0.3166	804.60	419.98	6.67	0.439	0.238	(2)
V870 Ara	0.082	1.503	1.67	0.3997	778.28	414.27	8.22	0.518	0.280	(3)
FP Boo	0.096	1.614	2.31	0.6405	673.88	365.01	9.73	0.460	0.249	(4)
DN Bootis	0.103	1.428	1.71	0.4476	709.84	399.05	7.59	0.439	0.238	(5)
CK Boo	0.107	1.442	1.521	0.3352	840.94	425.18	7.99	0.551	0.299	(4)
FG Hya	0.111	1.444	1.405	0.3278	792.26	442.69	6.96	0.514	0.279	(6)
GR Vir	0.122	1.376	1.49	0.347	788.40	419.64	7.34	0.492	0.266	(2)
$\epsilon$ CrA	0.128	1.72	2.12	0.5914	655.84	393.33	8.69	0.487	0.264	(6)
DZ Psc	0.135	1.352	1.469	0.3661	730.89	418.92	6.71	0.444	0.241	(2)
V776 Cas	0.138	1.75	1.821	0.4404	751.89	428.08	8.56	0.597	0.324	(7)
HN UMa	0.140	1.279	1.435	0.3825	681.55	412.26	6.11	0.385	0.209	(7)
V677 Cen	0.142	1.06	1.19	0.325	664.60	412.13	4.94	0.309	0.168	(8)
V410 Aur	0.144	1.304	1.397	0.3663	691.20	421.89	6.04	0.402	0.218	(9)
AH Cnc	0.149	1.21	1.36	0.3605	681.99	411.89	5.80	0.368	0.199	(10)
TZ Boo	0.153	0.72	0.97	0.2976	587.95	376.22	3.56	0.180	0.098	(6)

Columns: Stars-GCVS name of star;  $q$ -mass ratio;  $M_1$ -mass of the primary;  $R_1$ -radius of the primary;  $P$ -orbital period;  $v_e$ ,  $v_b$ -the equatorial velocity and the break-up velocity of the fast-rotating single star formed by the merger of W UMa binary in the case of the conservation of mass and angular momentum;  $R_{ex}$ -the radius of the expanded fast-rotating single star formed by the merger of W UMa binaries without mass loss;  $\delta M$  and  $\delta M_{mb}$ -the lost mass during during the merging process without and with considering the magnetic breaking.

References in Table 1: (1) Pribulla et al. 1999; (2) Gazeas et al. 2005; (3) Szalai et al. 2007; (4) Gazeas et al. 2006; (5) Şenavcı et al. 2008; (6) Yakut & Eggleton 2005; (7) Zola et al. 2005; (8) Maceroni & vantVeer 1996; (9) Yang, Qian & Zhu 2005; (10) Zhang, Zhang & Deng 2005.

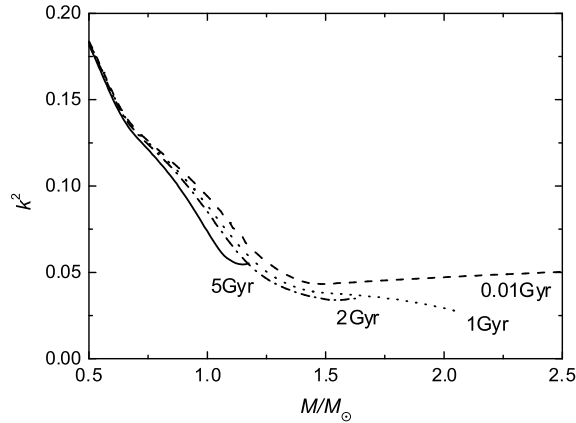
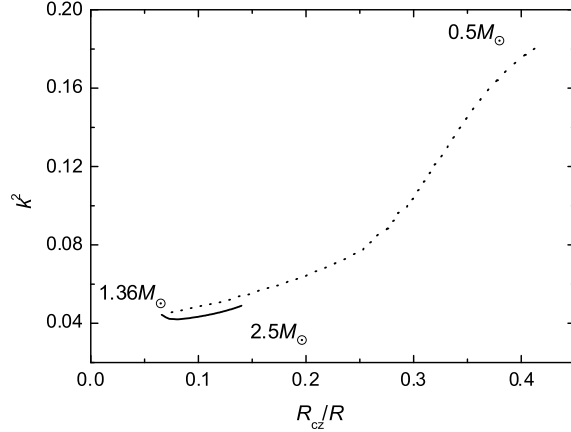


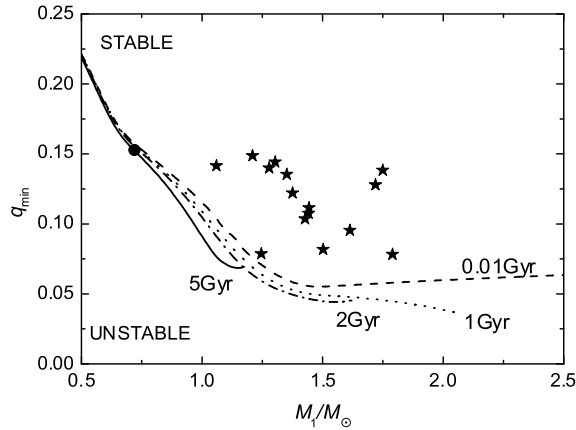
Figure 1. The relation of  $k^2$  vs  $M$  for stars at age=10Myr, 1Gyr, 2Gyr, and 5Gyr, respectively.

(1971, 1972) and Eggleton, Faulkner & Flannery (1973) and has been updated with latest input physics during the last three decades (e.g. Han et al. 1994; Pols et al. 1995, 1998; Nelson & Eggleton 2001; Eggleton & Kiseleva-Eggleton 2002). The observations of W UMa binaries show a well defined short-period limit of about 0.22 d (Rucinski 2008), which is equivalent to a lower mass limit for the primary of approximately  $0.6M_\odot$  (Stępień 2006). So we calculate the dimensionless gyration radii  $k$  of the stars with the solar metallicity ( $Z=0.02$ ) and with mass ( $M$ ) between  $0.5$  and  $2.5M_\odot$ .

Fig. 1 shows a relation of  $k^2$  vs  $M$  for the stars at age of 10Myr, 1Gyr, 2Gyr, and 5Gyr, respectively. It is seen from Fig.1 that  $k^2$  decreases with increasing age and this can



**Figure 2.** The relation of the fractional radius of the convection zone ( $R_{cz}/R$ ) and  $k^2$  for stars age=10Myr. The dotted line represents a relation between  $R_{cz}/R$  and  $k^2$  of the stars with a convective envelope and the solid line represents the relation between  $R_{cz}/R$  and  $k^2$  of the stars with a convective core. The change between convective envelope to core occurs at  $1.36M_{\odot}$ .



**Figure 3.** The relation of  $q_{min}$  and the mass of the primary of W UMa binary. Filled stars represent the observed W UMa binaries with extreme mass ratios and filled circle represents TZ Boo.

help explain the existence of low- $q$  W UMa systems as suggested by Li & Zhang (2006) and Arbutina (2007). In addition, we also found that  $k^2$  decreases with increasing mass of the star if the star's mass is less than about  $1.3M_{\odot}$ , and above this mass  $k^2$  is roughly constant. This might be caused by the difference in the structure of the main-sequence stars with different masses. We displays the relation of the fractional radius of the convection zone ( $R_{cz}/R$ ) and  $k^2$  for stars age=10Myr in Fig. 2 (where  $R_{cz}$  is the thickness of the convective zone and  $R$  the stellar radius). The dotted line represents a relation between  $R_{cz}/R$  and  $k^2$  of the stars with a convective envelope and the solid line represents the relation between  $R_{cz}/R$  and  $k^2$  of the stars with a convective core. It is found that for the stars with  $M \leq 1.36M_{\odot}$ , the convection zone is closed to the surface. If the mass increases from  $0.5M_{\odot}$  to  $1.36M_{\odot}$ , the fractional radius of the convection zone ( $R_{cz}/R$ ) decreases from 0.41 to 0.07 and the dimensionless gyration radius decreases from 0.18 to 0.045 with decreasing  $R_{cz}/R$ . For the

stars with  $M \geq 1.36M_{\odot}$ , the convection zone is closed to the center of star. The dimensionless gyration radius of the star is at a stable value of about 0.047 as  $R_{cz}/R$  increases from 0.066 to 0.14.

Li, Han & Zhang (2005) argued that the efficient energy transfer would decrease the dimensionless gyration radius of the secondary ( $k_2$ ) in a W UMa system and the value of  $k_2$  is not strongly different from that of the dimensionless gyration radius of the primary ( $k_1$ ), although there is a significant difference in the masses of the primary and the secondary. In fact, the radius of the secondary of each W UMa binary is much larger than that of a main-sequence star with the same mass and its mass distribution has been greatly changed compared with the main-sequence star due to energy transfer (Li, Han & Zhang 2005; Yakut & Eggleton 2005; Li et al. 2008). Rasio & Shapiro (1995) found that the dynamical stability limit of W UMa systems is at a contact degree ( $F$ ) of about 70 per cent ( $F = \frac{\Omega - \Omega_{in}}{\Omega_{out} - \Omega_{in}} \times 100\%$ , where  $\Omega$  is the surface potential of W UMa system,  $\Omega_{in}$  and  $\Omega_{out}$  are the potentials of the inner and the outer Lagrangian points, respectively). Therefore, assuming that the dimensionless gyration radii for both components are equal ( $k_1^2 = k_2^2$ ) and using the relation between the minimum mass ratio and  $k^2$  ( $F=0.7$ ) given by Li & Zhang (2006), we can obtain a relation between the theoretical minimum mass ratio and the mass of the primaries for W UMa systems, which is shown in Fig. 3. It is seen in Fig. 3 that the minimum mass ratio decreases with the evolutionary degree of the W UMa systems as suggested by Rasio (1995) and Li & Zhang (2006). This suggests that the dynamical stability limit of W UMa systems indeed depends on the evolutionary status of W UMa systems. It is also found that the minimum mass ratio of the young W UMa binaries with an age of 10 Myr decreases with increasing mass of the primary if the primary's mass is less than about  $1.3M_{\odot}$ , and above this mass the ratio is roughly constant.

We collected the physical parameters of some W UMa systems (listed in Table 1) with extreme mass ratios from the literature. The observed systems are also plotted in Fig. 3 with filled stars. TZ Boo is indicated with a different symbol (filled circle) because it has been found to be a quadruple system and its spectra has been contaminated by the third and fourth bodies (Pribulla et al. 2009). In addition, we do not include V857 Her, which has the lowest mass ratio of  $q=0.065$  (Qian et al. 2006), in our analysis as the primary mass is unknown. We predict its mass must be greater than  $1.25M_{\odot}$ . It is seen in Fig. 3 that W UMa systems are above the theoretical curves except for TZ Boo, i.e. they are located in the stable region although some of these systems have mass ratio lower than the minimum

mass ratio predicted by the previous theory. TZ Boo located in the unstable region might be due to the uncertainty of our stellar models that the metallicity effect is not considered. Another probable reason is the presence of the additional companion(s).

### 3 THE MERGER OF W UMa BINARIES

Li et al. (2008) argued that the mass ratio of W UMa systems become smaller and smaller owing to their dynamical evolution during their evolution. Therefore, they would merge into fast rotating stars due to Darwin's instability if their mass ratios have become smaller than the cutoff mass ratio of W UMa systems. When a W UMa binary begins to coalesce into a fast-rotating single star due to Darwin's instability, its orbital angular momentum can be expressed as  $J_{\text{orb}} = 3J_{\text{spin}}$  (Hut 1980; Eggleton & Kiseleva-Eggleton 2001). We assumed that the W UMa binary is in synchronous rotation (i.e.  $\omega_{\text{spin},1} = \omega_{\text{spin},2} = \omega_{\text{orb}} = \omega_0 = 2\pi/P_{\text{orb}}$ , where  $\omega_{\text{spin},1}$  and  $\omega_{\text{spin},2}$  are the spin angular velocities of two components and  $\omega_{\text{orb}}$  is the orbital angular velocity of the system). So the total angular momentum of a merging W UMa binary can be approximately expressed as

$$J_{\text{b,tot}} = J_{\text{orb}} + J_{\text{spin}} \approx 4J_{\text{spin}} = 4(k_1^2 M_1 R_1^2 + k_2^2 M_2 R_2^2)\omega_0. \quad (1)$$

where  $M_{1,2}$  and  $R_{1,2}$  are the masses and radii of the primary and the secondary in solar units, and  $k_{1,2}$  the dimensionless gyration radii for both components. We assumed that the dimensionless gyration radii for both components of W UMa binary are equal ( $k_1^2 = k_2^2$ ) as showed by Li, Han & Zhang (2005). When a W UMa binary with a mass ratio of  $q$  merges into a single star, we have

$$J_{\text{b,tot}} = \frac{4(1 + q^{1.92})}{1 + q} k_1^2 (M_1 + M_2) R_1^2 \omega_0, \quad (2)$$

where  $q (= M_2/M_1)$  is the mass ratio.

The angular momentum of the fast-rotating single star formed by the merger of W UMa binary can be written as

$$J_s = k^2 M R^2 \omega = k^2 M R v_e, \quad (3)$$

where  $M$  and  $R$  are the mass and radius of the fast-rotating single star formed by merger of the W UMa system in solar units,  $k$  the dimensionless gyration radius,  $\omega$ ,  $v_e$  the spin angular velocity and the equatorial velocity of the fast-rotating star formed from the merger.



### 3.1 The merger of W UMa binaries without angular momentum loss

We assumed that the merged star and the primary have the same radius and the same dimensionless gyration radius ( $k_1^2 \approx k^2$  and  $R_1 \approx R$ ) because the mass ratio of W UMa system is very low at the beginning of merger. If the total angular momentum and total mass are conserved in the course of the merger ( $J_s = J_{b,tot}$ ,  $M = M_1 + M_2$ ), using equation (2) and (3), we can get

$$\omega \approx \frac{4(1 + q^{1.92})}{1 + q} \omega_0, \quad (4)$$

and

$$v_e = R\omega \approx \frac{4(1 + q^{1.92})}{1 + q} R_1 \omega_0. \quad (5)$$

In addition, we can calculate the break-up velocities ( $v_b$ ) of the single stars formed by merger of W UMa systems if the W UMa systems with extreme mass ratios would merge into fast-rotating stars. The break-up velocity ( $v_b$ ) of a single star formed by the merger can be written as

$$v_b \approx \left( \frac{GM_1}{R_1} \right)^{\frac{1}{2}}. \quad (6)$$

Based on equations (5) and (6), the equatorial velocities and the break-up velocities for the single stars formed from the merger of W UMa systems are determined, and they are listed in Table 1. It is found that the distribution of the equatorial velocities of the fast-rotating single stars ranges from 588 to 819 km s<sup>-1</sup>. If the angular momentum and mass of W UMa systems are conserved in the course of merger, the single stars formed by the merger of W UMa binaries rotate at a velocity faster than their break-up velocities. This suggests that the applicability of the assumptions ( $J_s = J_{b,tot}$ ,  $R_1 \simeq R$ ) is unreasonable. In the course of the merger, W UMa systems should lose a large amount of mass and angular momentum, or the merged stars have expanded to a very large radius compared with that of a main-sequence star with the same mass.

It is noted that FK com stars are rapidly rotating G-type giants which might result from the merger of close (W UMa) binaries (Bopp & Rucinski 1981; Bopp & Stencel 1981; Webbink 1976). We assumed that the fast-rotating star expands to a very large radius ( $R = R_{ex} \gg R_1$ ) and  $J_s = J_{b,tot}$ . Using equation (2) and equation (3), we can calculate the radius of the expanded merged star formed by the merger of W UMa binary

$$R_{ex} \approx \frac{4(1 + q^{1.92})}{1 + q} \frac{R_1^2 \omega_0}{v_e}. \quad (7)$$

It is found that the radius of the expended single star depends on the equatorial velocity. Rucinski (1990) showed that the rotational velocity ( $v \sin i$ ) of FK Com star is about  $160 \text{ km s}^{-1}$ . So we assumed that the equatorial velocity of the expanded merged star is  $160 \text{ km s}^{-1}$ . Then we found that the expanded merged stars formed by the merger of W UMa binaries have the radii of about  $3.6 \sim 9.7 R_\odot$  (listed in table 1) and are 3.7~5.3 times the radii of the primaries.

### 3.2 The mass loss without the magnetic braking

If the merged stars do not expand, W UMa systems should lose a large amount of mass and angular momentum in the course of the merger. The mass loss ( $\delta M$ ) during the merging process can be calculated by using the following formulae:

$$J_{\text{b,tot}} = k_s^2 (M_1 + M_2 - \delta M) R_s v_s + \delta M R_{\text{esc}} v_{\text{esc}}, \quad (8)$$

where  $k_s$ ,  $R_s$  and  $v_s$  are the dimensionless gyration radius, the radius and the equatorial velocity of the merged star;  $R_{\text{esc}}$  and  $v_{\text{esc}}$  are the radius and the velocity of the stellar wind escaped from the system. We assumed that the merged star and the primary have the same radius and the same dimensionless gyration radius ( $k_1^2 \approx k_s^2$  and  $R_1 \approx R_s$ ). De Marco et al. (2005) gave a mean value of the rotational velocity for five fast rotating blue stragglers to be of about  $160 \text{ km s}^{-1}$ . So we assumed that the equatorial velocity of the merged star is  $v_{\text{re}} \approx 160 \text{ km s}^{-1}$ . If the effect of magnetic braking is not considered, we can take that  $R_{\text{esc}} \approx R_s \approx R_1$  and  $v_{\text{esc}} \approx v_s \approx 160 \text{ km s}^{-1}$ . Then, using equations (2) and (8), we obtain

$$\delta M = \left[ \frac{4(1 + q^{1.92})}{1 + q} \frac{R_1 \omega_0}{160} - 1 \right] \frac{k^2}{1 - k^2} (M_1 + M_2). \quad (9)$$

We assumed  $k^2$  to be about 0.075. Based on equation (9), the lost mass during the merging process of W UMa systems can be calculated and listed in Table 1. It is found that the distribution of the mass loss has a range from  $0.18 \sim 0.65 M_\odot$ , which is about 21 ~ 33 per cent of the total mass.

### 3.3 The mass loss with the magnetic braking

The stars which have convective envelope can be magnetically braked and slowed down faster than just angular momentum loss in stellar wind (Tout & Pringle 1992). The stellar wind is forced by the magnetic field to corotate out to the Alfvén radius ( $R_A$ ) then escapes freely ( $R_{\text{esc}} = R_A$ ). Then the same mass lost from the systems would take away more angular

momentum than that escaped from the surface of the systems. We studied the mass loss ( $\delta M_{mb}$ ) in the merging process of W UMa binary when the effect of magnetic braking is included. We assumed that the stellar wind is in synchronous rotation with the merged stars ( $v_{\text{esc}}/R_A = v_s/R_s$ ) and  $v_s \simeq 160 \text{ km s}^{-1}$ . Then, the mass loss can be written as

$$\delta M_{mb} = \left[ \frac{4(1 + q^{1.92})}{1 + q} \frac{R_1 \omega_0}{160} - 1 \right] \frac{k^2}{\left(\frac{R_A}{R_1}\right)^2 - k^2} (M_1 + M_2). \quad (10)$$

The Alfvén radius can be expressed as (Tout & Pringle 1992):

$$\frac{R_A}{R_1} = 1.1 f^{-1/4} (\gamma/10^{-2})^{1/2}, \quad (11)$$

in which

$$f = \omega/\omega_b, \quad (12)$$

where  $\gamma$  is the efficiency of dynamo regeneration ( $\sim 10^{-2}$ ),  $\omega$  and  $\omega_b$  are the angular velocity and the break-up angular velocity of the star. The break-up angular velocity reads

$$\omega_b = \left( \frac{GM_1}{R_1^3} \right)^{1/2}, \quad (13)$$

and the angular velocity reads

$$\omega = \frac{G^{1/2} (M_1 + M_2)^{1/2}}{A^{3/2}}. \quad (14)$$

Based on equations (12), (13) and (14), we can obtain

$$f = (1 + q)^{1/2} \left( \frac{0.49 q^{-2/3}}{0.6 q^{-2/3} + \ln(1 + q^{-1/3})} \right)^{3/2}. \quad (15)$$

If  $q \sim 0.1$ , we can obtain

$$f \approx 0.4611. \quad (16)$$

From equation (11) and equation (16), we find

$$\frac{R_A}{R_1} \approx 1.335. \quad (17)$$

Hence, the mass loss ( $\delta M_{mb}$ ) is determined by using equation (10) and equation (17) with considering the effect of magnetic braking and listed in Table 2. The distribution of the mass loss has a range from  $0.10 \sim 0.35 M_\odot$  in the course of merger which is about  $12 \sim 18$  per cent of the total masses.

## 4 DISCUSSION AND CONCLUSIONS

In this paper, we investigated the minimum mass ratio of W UMa systems that have different primary masses. In addition, we studied the mass loss during the merger of W UMa systems.

Arbutina (2009) has investigated the theoretical stability limit of W UMa binary, where he has considered the effects of rotation which would increase the central concentration. He gave the minimum mass ratio of W UMa binaries to be 0.070-0.074. Considering the different structure of the primaries, we found that the minimum mass ratio of the young W UMa binaries with an age of 10 Myr decreases with increasing mass of the primary if the primary's mass is less than about  $1.3M_{\odot}$ , and above this mass the ratio is roughly constant. The minimum mass ratio of binaries with  $M_1 \geq 1.3M_{\odot}$  is lower than the value given by Arbutina (2009). This is mainly because he considered the effect of rotation based on  $n = 3$  polytrope (which has  $k^2 \approx 0.075$ ). By comparing the theoretical minimum mass ratio with the observational data, it is found that the observational systems are in the stable region except for TZ Boo. This means that these observed W UMa systems are dynamically stable and the existence of low- $q$  systems can be explained by the different structure of the primaries with different masses. This suggests that the dimensionless gyration radius and thus the structure of the primary is very important in determining the minimum mass ratio. A W UMa system with a less massive primary will merge at a larger mass ratio. Therefore, it is necessary to consider the different structure of the primaries for the study of the dynamical stability of W UMa systems.

Previous theoretical studies have argued that W UMa binary would eventually merge into a single star (Webbink 1976, 1985; Tutukov & Yungelson 1987; Mateo et al. 1990). Assuming that the angular momentum and mass are not lost from W UMa systems during the merging process, it is found that the merged stars rotate faster than their break-up velocities, which is unreasonable. One possible explanation is the merged stars expand to a very large radius which is about be 3.7~5.3 times the radii of the primaries. These expanded merged stars can be observed like FK com-type stars. We need to obtain the parameters of FK Com-type stars and make comparisons with our results.

Another possible explanation is that during the merger, W UMa systems should lose a large amount of mass and angular momentum. Chen & Han (2008) predicted that a large amount of mass ( $\sim 0.5M_{\odot}$ ) must be lost from W UMa systems in the course of merger if the theoretical model can match the observations. However, they did not give a physical mechanism for the mass loss. We calculated that the mass loss during the merging process of W UMa system would be of 21~33 per cent of the total mass. If the effect of magnetic braking is considered, the angular momentum loss due to mass loss may be more efficient and the mass loss would decrease to be 12~18 per cent of their total masses. Our results

are smaller than that predicted by Chen & Han (2008). This might be due to the lack of W UMa systems with  $M_1 \geq 2.0M_\odot$  in our sample and these systems would lose more mass during the merging process.

There is a significant difference between the theoretical prediction and the observations of the rotation velocities of the blue stragglers and FK Com-type giants. In the color-magnitude diagrams of globular clusters, some W UMa systems are, in fact, in the region of the blue stragglers, and there are 20 W UMa binaries observed among about 900 blue stragglers (Von Braun & Mateo 2002; Von Braun 2003; Rucinski 2000, and references therein). This implies that the formation of some blue stragglers is related to the merger of W UMa systems. In addition, Taam & Sandquist (2000) suggested that binaries in the common envelope phase at short orbital periods must eventually merge into a single star since the massive component cannot be on the giant branch. Our study indicates that the significant angular momentum and mass might be lost from W UMa system in the course of the merging process, and this kind of mass and angular momentum loss might be driven by the release of orbital energy of the secondaries. This is similar to common-envelope evolution. When the secondary of W UMa system with high spin velocity merges into the primary, the orbital energy is deposited into the envelope, disrupting it. This energy can only make some mass of the envelope to be ejected, because for the stars near the main sequence, the binding energy of the envelope is too large for energy from the orbital motion to completely eject it (Taam & Sandquist 2000). The other possible mechanism are first a circum-stellar disk. The mass lost from the system might form a circum-stellar disk since the the equatorial rotational velocity is the fastest. This circum-stellar disk would extract the angular momentum from the single stars formed by the merger of the W UMa systems through the tidal torque (Chen, Li & Qian 2006). Second, the angular momentum loss is caused by W UMa systems' companion(s) through tidal friction. The progenitors (W UMa binaries) of some blue stragglers and FK Com-type giants might be formed in primordial and dynamically formed triple systems (Leonard 1996; Perets & Fabrycky 2009). Pribulla & Rucinski (2006) showed that most W UMa binaries exist in multiple systems. The presence of distant companions can facilitate not only the formation of W UMa binaries but also the deceleration of the fast-rotating single stars formed by the merger of W UMa binaries. Third, for FK Com-type giants, their radii are much larger than that of the main-sequence stars with the same masses and their dimensionless gyration radii have risen to a higher value because their envelopes have become convective. It would undertake the decrease in the rotation velocities of FK Com-type giants.

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